Standard Model Implementation

Recall the standard model is specified by a sample space, Ω, a set of strictly increasing times, , a collection of increasingly fine finite partitions of Ω, , prices , and cash flows , where is the number of market instruments in the model.

A trading strategy is a finite collection of strictly increasing stopping times, , and trades, indicating the number of shares to trade in each of the instruments. Trades accumulate to a position, where when .

The value of a position at time is and the amount generated by the trading strategy at time is .

A model is arbitrage-free if there is no closed-out trading strategy () with and for . The Fundamental Theorem of Asset Pricing states this is the case if and only if there exists a positive adapted process, , with

A simple corollary using the definition of value and amount shows

For a closed-out strategy, . Since and we have using is positive, where the 0 subscript denote time .

Every model of the form where is a martingale and is a positive adapted process is arbitrage-free.

Define the stopping time then